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ELECTROMAGNETIC FLOWMETER SENSITIVITY WITH TWO-PHASE FLOW

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Abstract—The behaviour of the uniform-field, point-electrode electromagnetic flowmeter with two-phase flows is discussed. It is emphasized that, in general, the signal is dependent on the velocity profile and on the degrees of uniformity and isotropy of the suspension. The weight function distribution of the annular flowmeter is described.

1. INTRODUCTION

The use of the electromagnetic flowmeter in a two-phase flow was discussed recently by Bernier & Brennen (1983). They concluded that calibration of the flowmeter is both quite independent of the flow regime and quite insensitive to the distribution of the disperse phase. These conclusions are not in general correct, even when the flow is rectilinear and axisymmetric. The purpose of this paper is to clarify the position and to draw attention to other relevant work.

2. THEORY

The three-dimensional theory of electromagnetic flowmeters given by Bevir (1970, 1971a) and discussed by Wyatt (1977, 1984) is summarized as follows. The potential between two electrodes in a conducting medium which includes a liquid volume specified by τ in which there is a velocity distribution $\mathbf{v}$ and a magnetic field $\mathbf{B}$ is

$$U = \int_\tau \mathbf{W}\cdot\mathbf{v}\,\mathrm{d}\tau; \qquad [1]$$

$\mathbf{W}$ is the *weight vector*—it weights the contribution to U due to the velocity $\mathbf{v}$ at every point. $\mathbf{W}$ is given by

$$\mathbf{W} = \mathbf{B} \times \mathbf{j}, \qquad [2]$$

where $\mathbf{j}$ is a hypothetical current density known as the *virtual current*; $\mathbf{j}$ is the current density that would be set up in the stationary liquid by passing unit current into one electrode and extracting it from the other. When $\mathbf{j}$ is formulated, it defines mathematically the electrodes, the flowmeter boundary and the conductivity distribution within that boundary. $\mathbf{j}$ weights the effect at the electrodes of elemental $\mathbf{B} \times \mathbf{v}$ generators at every point in the liquid. In principle, the concept of virtual current permits any flowmeter problem to be solved; it also clarifies the functioning of electromagnetic flow and velocity measuring devices.

3. THE CIRCULAR, UNIFORM-FIELD, POINT-ELECTRODE FLOWMETER

Figure 1 illustrates this flowmeter with a cylindrical coordinate system (r, θ, z). The electrodes are situated at $(b, \pm\pi/2, 0)$, where b is the pipe radius. The magnetic field of

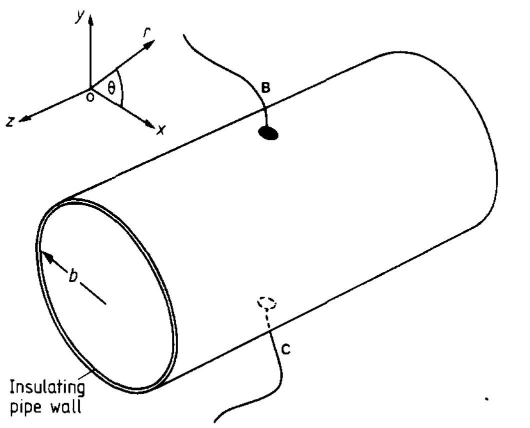

Figure 1. The circular, uniform-field flowmeter with "point" electrodes at B and C.

strength B, lies in the direction Ox. We assume that the flow is *rectilinear*, i.e. $\mathbf{v} = [0, 0, v(r, \theta)]$. Then [1] reduces to

$$U = \int_0^{2\pi} \int_0^b W(r, \theta)\, v(r, \theta) r \, \mathrm{d}r \, \mathrm{d}\theta. \qquad [3]$$

Here $W(r, \theta)$ is the *rectilinear weight function*, given by

$$W(r, \theta) = \int_{-\infty}^{\infty} W_z \, \mathrm{d}z, \qquad [4]$$

where

$$W_z = B_r j_\theta - B_\theta j_r. \qquad [5]$$

When the flow is *axisymmetric* as well as rectilinear, $\mathbf{v} = [0, 0, v(r)]$ and [3] becomes

$$U = 2\pi \int_0^b W'(r)\, v(r) r \, \mathrm{d}r, \qquad [6]$$

where the *axisymmetric weight function* $W'(r)$ is given by

$$W'(r) = \frac{1}{2\pi} \int_0^{2\pi} W(r, \theta) \, \mathrm{d}\theta. \qquad [7]$$

W_z can be written in rectangular coordinates (x, y, z) as

$$W_z = B_x j_y - B_y j_x. \qquad [8]$$

If the field is uniform, $B_x = B =$ constant and $B_y = 0$. Since j_y is harmonic it follows that W_z and therefore $W(r, \theta)$ are harmonic also. The mean-value theorem for harmonic functions immediately gives $W'(r) =$ constant (Bevir 1971a) whence, from [6], U is proportional to the flowrate and independent of the velocity distribution when this is axisymmetric. The magnitude of the signal obtained from the uniform field flowmeter with axisymmetric flow can be most simply found by considering the simplest axisymmetric flow, namely that due to a uniform rectilinear velocity field $\mathbf{v}$. Since div $\mathbf{B}$ is zero, the line integral $\oint \mathbf{B} \times \mathbf{v} \cdot \mathbf{dl}$ is zero also. Therefore, provided the electrodes are of infinitely small area, no currents flow in the liquid. Hence

$$U = \int_{\mathrm{C}}^{\mathrm{B}} \mathbf{B} \times \mathbf{v} \cdot \mathbf{dl}, \qquad [9]$$

where the line integral is taken from electrode C to electrode B along any path. Thus.

$$U = 2Bvb = \frac{2B}{\pi b} Q, \qquad [10]$$

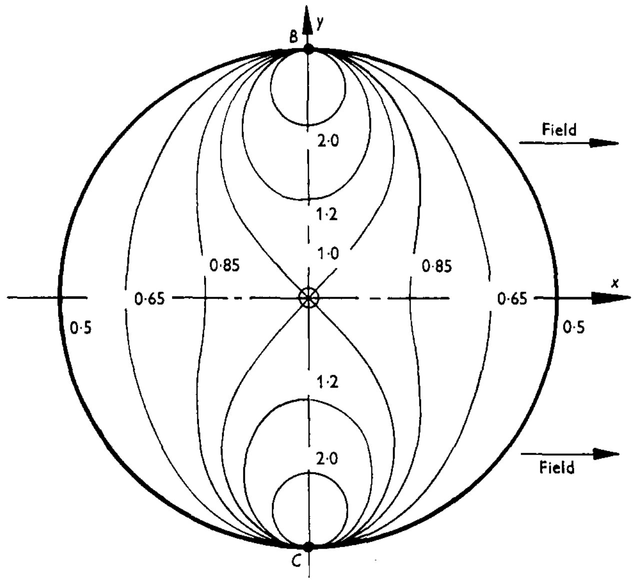

Fig. 2. Normalized rectilinear weight function $W\pi b/2B$ of a uniform-field flowmeter with point electrodes (Shercliff 1962).

where Q is the flowrate. This is the relationship between signal and flowrate for any axisymmetric flow. The independence of sensitivity (signal for unit flowrate) on the axisymmetric velocity profile is known as the *axisymmetric property*.

The response of the flowmeter to rectilinear flow at various points in the cross-section varies widely, even when the axisymmetric property holds. The response is found by determination of $\mathbf{j}$ with the given boundary conditions and substitution of this in [5] and [4]. This yields Shercliff's weight function (Shercliff 1962):

$$W(R,\theta) = \frac{2B}{\pi b}\frac{1 + R^2\cos 2\theta}{1 + 2R^2\cos 2\theta + R^4}, \qquad [11]$$

where $R = r/b$. $W(R,\theta)$ is shown in figure 2.

4. ANNULAR FLOW

Figure 3 illustrates a uniform-field flowmeter with a stationary coaxial insulating core, which permits flow in the annulus only. It can be shown, by means of Cauchy's integral formula, that $W'(r)$ is uniform in the annulus, i.e. the axisymmetric property holds for this flowmeter also. If the electrodes are points, the sensitivity can easily be found. We consider the conventional flowmeter with a uniform velocity field. Since in this case no actual currents flow, introduction of the insulating coaxial core will not affect the flowmeter response with a given velocity. That is, the signal remains the same although the flowrate is reduced by the factor $(b^2 - a^2)/b^2$, whence

$$U = \frac{2B}{\pi b} Q \frac{b^2}{b^2 - a^2}. \qquad [12]$$

This result† holds for any axisymmetric velocity profile and is in agreement with the result quoted by Bernier & Brennen (1983).

†Equation [12] follows rigorously from [9], which applies in the annulus when $\mathbf{v}$ is rectilinear and uniform. The line integral is taken along a convenient path in the annulus.

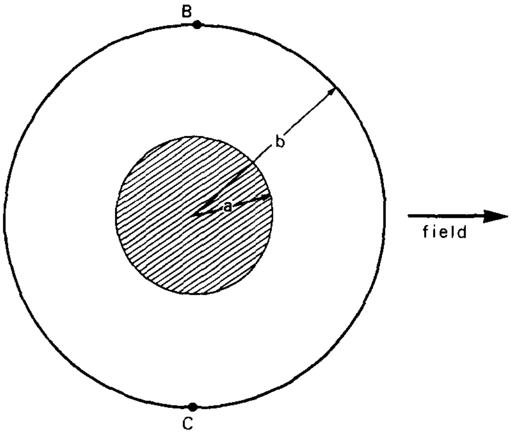

Figure 3. Section of a flowmeter with a coaxial, insulating core.

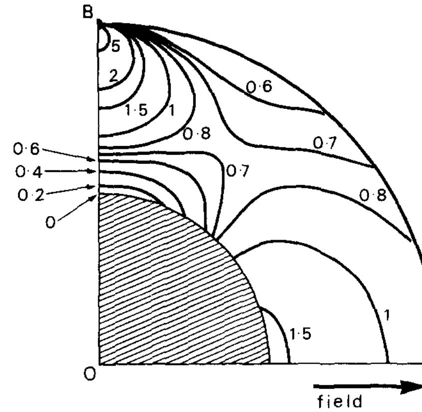

Figure 4. Normalized rectilinear weight function $(1-\alpha)W\pi b/2B$ of a uniform-field, annular flowmeter with point electrodes, $a/b = 0.5$. The shaded region is insulating. One quadrant only of the cross-section is shown, with an electrode at B. The curves in the other quadrants are mirror-images (cf. figure 2).

Equation [12] holds if the core consists of a long gas bubble moving with any velocity. In this case we can consider the system as one of two-phase flow, in which the void fraction is $\alpha = a^2/b^2$. The signal can be written

$$U = \frac{2B}{\pi b}\frac{Q_L}{1-\alpha}; \tag{13}$$

Q_L is identical with Q in [12], but the subscript emphasizes that the flow referred to is that of the liquid phase.

Here also the response to rectilinear flow at different points in the annulus varies widely. We can show, by finding the virtual current with the given boundary conditions, that

$$W(R,\theta) = \frac{2B}{\pi b}\left\{\sum_{m=0}^{\infty}(-1)^m R^{2m}\cos 2m\theta + \sum_{m=0}^{\infty}(-1)^m A_m\left[R^{2m}\cos 2m\theta + R^{-2(m+1)}\cos 2(m+1)\theta\right]\right\} \tag{14}$$

where $R = r/b$, $A_m = \alpha^{2m+1}/(1-\alpha^{2m+1})$ and $\alpha = a^2/b^2$. The first series is Shercliff's weight function in series form $(R < 1)$. Integration of $W(R,\theta)$ to give $W'(R)$ yields $(2B/\pi b)b^2/(b^2-a^2)$, in agreement with [12]. $W(R,\theta)$ is illustrated in figure 4. The weight function is zero at the point where the line BO (which passes through the electrodes) intersects the insulating core, because at this point $\mathbf{j}$ is zero. Near this point, $\mathbf{j}$ is nearly parallel to the field, so $\mathbf{B}\times\mathbf{j}$ is relatively small.

It is evident that the narrower the annulus, the more uniform the virtual current will be in regions distant from the electrodes. Also, since $\mathbf{B}$ is constant and directed as shown in figure 4, we would expect the magnitude of $\mathbf{W}$ to vary approximately as $\cos\theta$, except near the electrodes. These features are demonstrated in figure 5.

5. CORE FLOW

Annular flow with an insulating core (section 4) is, strictly speaking, a two-phase flow in which the insulating disperse phase is entirely separated from the liquid phase. The "reverse" of this system is shown in figure 6, which illustrates a flowmeter in which the interior surface of the wall is bounded by a stationary, poorly-conducting concentration of disperse phase, with a well-conducting liquid phase flowing within it. The signal, for

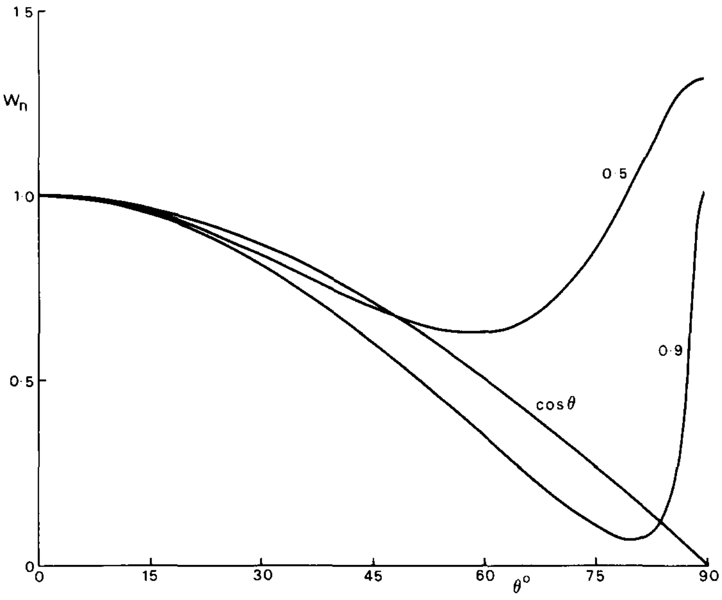

Figure 5. Normalized rectilinear weight function W_n of a uniform-field, annular flowmeter with point electrodes, along an arc midway between the circles bounding the annulus, normalized to 1 at $\theta = 0$. $a/b = 0.5$ and 0.9 (see the text).

any axisymmetric velocity profile, is (Wyatt 1968)

$$U = \frac{2B}{\pi b} Q_L \frac{2\dfrac{\sigma_L}{\sigma_D}}{2\dfrac{\sigma_L}{\sigma_D} - \left(1 - \dfrac{a^2}{b^2}\right)\left(\dfrac{\sigma_L}{\sigma_D} - 1\right)}; \qquad [15]$$

σ_L and σ_D are, respectively, the conductivities of the liquid- and disperse-phase concentrations. If $\sigma_L/\sigma_D \to \infty$ and the void factor $(b^2 - a^2)/b^2 = \alpha$, then

$$U = \frac{2B}{\pi b} \frac{Q_L}{1 - \dfrac{\alpha}{2}}. \qquad [16]$$

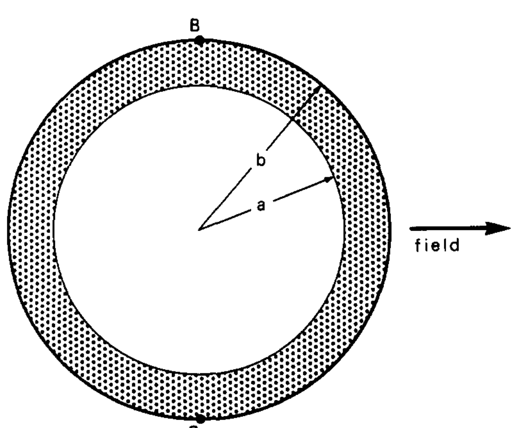

Figure 6. Section of a flowmeter illustrating core flow. The dotted area represents the stationary, concentrated disperse phase.

6. SUSPENSION FLOW

Consider the virtual current distribution in a stationary, uniform, isotropic suspension of equal insulating spheroids which are small in comparison with the flowmeter radius. Because the suspension is uniform and isotropic, the virtual current distribution, macroscopically speaking, is not altered by the presence of the spheroids. That is, the virtual current entering and leaving any volume of the suspension, large in comparison with a spheroid, is not altered. Provided the uniformity of the suspension is maintained when the fluid is in motion, the contribution of such a volume to the flowmeter integral [1], will, for a given velocity distribution within it, be unchanged and the presence of the spheroids will be undetected by the flowmeter. What actually happens, microscopically speaking, is that the reduced volumes of liquid between spheroids cause an increase in local mean virtual current density; but this is exactly balanced by the reduced volume of liquid within which it occurs, so the contribution to the volume integral remains unchanged for a given local velocity and magnetic field.

If local circulations occur in the suspension when it is flowing, the number of spheroids in a given volume will vary. This will lead to local variations in virtual current and therefore to variations in the contribution of a given volume of suspension to the volume integral. We therefore need to specify some minimum time δt over which the response of the flowmeter is averaged in order to eliminate the effect of such variations. Otherwise noise will be observed superimposed on the flowmeter signal, in addition to that due to the local velocity variations themselves.

When the suspension flows, it is possible for forces due to velocity gradients to create a non-uniform distribution of the spheroids (cf. Segré & Silberberg 1961; Takano *et al.* 1968). This will always alter the virtual current distribution and will in general change the flowmeter signal for a given flowrate.

If the suspended medium consists of irregularly-shaped bodies instead of spheroids, but which still have a maximum dimension small in comparison with the flowmeter radius, and which are both uniformly distributed and randomly oriented, it will be clear that we still have, macroscopically speaking, a uniform, isotropic suspension. When the suspension flows the particles will, in general, tumble and rotate and, if not rigid, deform (Goldsmith 1967); but provided δt is sufficiently long, they will not have an effect different from that of spheroids. However, particles which are not spheroidal, but which are of regular and uniform shape, may interact hydrodynamically with the liquid in such a way that they are aligned relative to the flowmeter axis for a disproportionately large part of their rotational period. They may thus create an effectively non-isotropic, as well as a non-uniform, suspension. This can further alter the effective virtual current distribution and therefore change the flowmeter signal.

Note that nothing that has been said is incompatible with a local relative velocity, or "slippage", between the particles and the liquid.

7. A UNIFORM, ISOTROPIC SUSPENSION

It is evident from the preceding section that the signal for this case is

$$U = \frac{2B}{\pi b}\frac{Q_L}{1-\alpha}. \qquad [17]$$

8. A UNIFORM SUSPENSION OF LONG, THIN RODS PARALLEL TO THE FLOW

A uniform suspension of infinitely long, thin, insulating rods which are aligned in a rectilinear flow parallel to the flowmeter axis is electrically anisotropic. However, in the case of the uniform-field flowmeter this anisotropy has no effect on the signal. The reason is that the uniform field flowmeter is a two-dimensional device, i.e. it is infinitely long in the direction of its axis. When the flow is rectilinear, no actual currents flow *parallel to* *the axis*, so that, apart from increasing the void factor, the *length* of the rods has no effect. Looked at another way, the effective electrodes in a "long" flowmeter are two long strips parallel to the axis which pass through the actual electrodes (Bevir 1971a; Wyatt 1984). The virtual current of these electrodes flows only in planes normal to the axis. Therefore, provided the maximum dimension in the cross-section of the rods is small and they are randomly oriented about their axes, the suspension seen by the virtual current in such planes is macroscopically uniform and isotropic. Again then,

$$U = \frac{2B}{\pi b}\frac{Q_{\mathrm{L}}}{1-\alpha}. \qquad [18]$$

9. A UNIFORM SUSPENSION OF THIN DISCS ALIGNED PARALLEL TO THE FLOW

Dennis & Wyatt (1969) observed that the sensitivity of a quasi-uniform field electromagnetic flowmeter to the flow of blood was not strictly independent of the haematocrit (concentration of red blood cells, or erythrocytes). That is to say, the signal did not strictly obey the equation

$$U = \frac{2B}{\pi b}\frac{Q_{\mathrm{L}}}{1-\alpha}.$$

They found that the flowmeter sensitivity depended upon both the haematocrit and the flow regime, even when correction was made for the non-uniformity of the magnetic field. They ascribed their results to the non-uniform and/or anisotropic distribution of the erythrocytes. Figure 7 shows a cross-section of the flowmeter in the electrode plane, illustrating sections of uniformly-distributed erythrocytes, approximated by thin discs aligned parallel to the (laminar) flow. The figure demonstrates the anisotropy of the electrical conductivity of the suspension, due to the difference in the geometrical projections of the discs seen in the radial and circumferential directions, respectively. The anisotropy will alter the virtual current distribution and therefore the weight function distribution and sensitivity of the flowmeter. In practical terms, when the flow is non-uniform (but still axisymmetric) the induced currents that actually flow in the cross-section when the flowmeter is working will be altered by the anisotropy, leading to a difference in signal. When the flow is turbulent, it is likely that the erythrocytes tumble, destroying the anisotropy (see section 6).

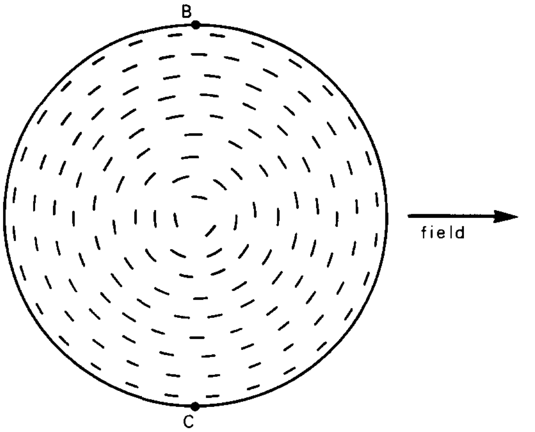

Figure 7. Cross-section of a flowmeter in which there is a uniform suspension of thin discs which have their main surfaces aligned with the rectilinear flow.

10. THE THEORY OF FLOWMETER RESPONSE WITH NON-UNIFORM AND NON-ISOTROPIC SUSPENSIONS

The theory has been given by Bevir (1971b). We quote, as one example of the effects that can occur, the result for a suspension with a non-uniform, isotropic conductivity distribution which varies with radius r $(0 < r < 1)$ in accordance with the power law

$$\sigma(r) = 1 + \lambda r^k, \quad [19]$$

and in which there is an axisymmetric velocity profile

$$v \propto 1 - r^n. \quad [20]$$

We give Bevir's result for the signal (his equation [30], quoted here in unnormalized form):

$$U = \frac{2B}{\pi b} \frac{Q_L}{1-\alpha} \left[1 - \frac{\lambda k}{(k+2)(k+n+2)} \right]. \quad [21]$$

This shows that U is dependent both on the conductivity distribution (λ, k) and on the velocity profile (n). Bevir also gives results for anisotropic, non-uniform conductivity distributions together with non-uniform, axisymmetric velocity profiles. An example of this applies to the uniform, anisotropic suspension illustrated in figure 7, for which Bevir gives the result (his equation [20], quoted here in unnormalized form):

$$U = \frac{2B}{\pi b} \frac{Q_L}{1-\alpha} \frac{1}{1 + \dfrac{\gamma - 1}{n+2}} \quad [22]$$

where $\gamma^2 = \sigma_\theta/\sigma_r$. σ_θ and σ_r represent the conductivities of the suspension measured in the circumferential and radial directions, respectively; σ_θ and σ_r are assumed to be different but independent of r. Here we see that U is dependent both on the anisotropy (γ) and on the velocity profile (n).

11. DISCUSSION

The mean velocity of the continuous (liquid) phase in a suspension, whether or not the disperse (suspended) phase is uniformly distributed, is $Q_L/\pi b^2\,(1-\alpha)$, where Q_L is the liquid flowrate and α is the void (disperse) fraction. Bernier & Brennen noted the equality of [10], [13], [17] and [18] (in their case, the near-equality of [18]). This, together with their experimental results, led them to state (see section 1): "... the [uniform-field] electromagnetic meter... does not only measure the mean velocity of the continuous phase... but this measurement appears to be quite insensitive to the distribution of the disperse phase". Also (see the abstract) they state that "... the calibration is quite independent of... axisymmetric velocity profile... ". These inferences are correct when the disperse phase consists of uniformly-distributed, small, randomly-oriented particles, which create a macroscopically uniform, isotropic suspension. They are not in general correct when the suspension is isotropic but non-uniform (see [13], [16], [17] and [21]); or when the suspension is uniform but anisotropic (see [22]); or when the suspension is both non-uniform and anisotropic (Bevir 1971b).

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